Name:_____ Date:_____

Pre Calculus 11: HW Section 1.6 Sigma Notations and Summation

1. Indicate the number of terms in each series. Determine whether if it is an arithmetic or geometric series

a) $\sum_{x=1}^{11} 2x+1$ n=11-1 +1= 11 terms Arithmetic	b) $\sum_{x=2}^{13} 3(2)^{x-5}$ n=13-2+1= 13 terms Geometric	c) $\sum_{x=1}^{99} 2^x+3$ n=99-1+1= 99 terms Geometric	d) $\sum_{x=2}^{11} 7x-4$ n=11-2+1= 10 terms Arithmetic
e) $\sum_{x=3}^{20} 4(x-1)+3$ n=20-3+1=18 terms Arithmetic	f) $\sum_{x=n-2}^{n+6} x+7$ n=n+6-(n-2)+1=9 terms Arithmetic	g) $\sum_{x=a}^{9} 2x=84$ arithmetic a=3, so 9-3+1=7 terms a=-2, so 9+2+1=12 terms	h) $\sum_{x=3}^{a} x^2=814$ $3^2+4^2+5^2+6^2+7^2+8^2+$ $9^2+10^2+11^2+12^2+13^2=814$ a=13, so 13-3+1=11terms Neither Geometric or Arithmetic

2. Write the series corresponding to each expression and then find the sum. Please show all your work!

a) $\sum_{x=4}^{12} 2(3+x)$

$=2(3+4)+2(3+5)+2(3+6)+....+2(3+12)$

$a=14$

$l=45$

$n=12-4+1=9$

$S=\frac{n}{2}(a+l)$

$S=\frac{9}{2}(14+30)$

$S=198$

b) $\sum_{x=2}^{8} x-4$

$(2-4)+(3-4)+(4-4)+...(8-4)$

$a=-2$

$l=4$

$n=8-2+1=7$

$S=\frac{n}{2}(a+l)$

$S=\frac{7}{2}(-2+4)$

$S=7$

c) $\sum_{x=-2}^{4} 3^x$

$3^{-2}+3^{-1}+3^0+3^1+...3^4$

$a=\frac{1}{9}$

$r=3$

$n=4-(-2)+1=7$

$S=\frac{a(r^n-1)}{r-1}$

$S=\frac{\frac{1}{9}(3^7-1)}{2}$

$S=121.\overline{4}$

d) $\sum_{x=5}^{10} x^2$

$5^2+6^2+7^2+8^2+9^2+10^2$

This is neither gecometric nor arithmetic. Best way to find the sum is to add them up

$25+36+49+64+81+100$

$=355$

e) $\sum_{x=-5}^{9} 3^{x-2}$

$3^{-7}+3^{-6}+3^{-5}+3^{-4}+...3^{7}$

$a=3^{-7}$

$r=3$

$n=9-(-5)+1=15$

$S=\frac{a(r^n-1)}{r-1}$

$S=\frac{3^{-7}(3^{15}-1)}{3-1}$

$S=\frac{3^8-3^{-7}}{2}=\frac{6561-0.00045724}{2}$

$S=3280.4998$

f) $\sum_{x=1}^{6} 5(2)^{x-1}$

$5(2)^0+5(2)^1+5(2)^2+5(2)^3+...5(2)^5$

$a=5$

$r=2$

$n=6-(1)+1=6$

$S=\frac{a(r^n-1)}{r-1}$

$S=\frac{5(2^6-1)}{2-1}$

$S=315$

3. Express each series using sigma notations. Please show all your work

a) $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$

Method 1:

There are 7 terms, n=7: Rewrite the sequence

$2^1+2^0+2^{-1}+2^{-2}+2^{-3}+2^{-4}+2^{-5}$

With the exponents going down, rewrite it

$2^{-(-1)}+2^{-(0)}+2^{-(1)}+2^{-(2)}+2^{-(3)}+2^{-(4)}+2^{-(5)}$

So your function will look like this:

$f(x)=2^{-x}$, with *"x"* starting with -1 and end with 5

$\sum_{x=-1}^{5} 2^{-x}$

Method 2:

Rewrite the sequence:

$\frac{1}{32}+\frac{1}{16}+\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+1+2$

$2^{-5}+2^{-4}+2^{-3}+2^{-2}+2^{-1}+2^0+2$

Your function will look like this:

$f(x)=2^x$, with *"x"* starting with -5 and end with 1

$\sum_{x=-5}^{1} 2^{x}$

b) $3+7+11+15+19+23+27+31+35$

This is an arithmetic sequence:

$3+(3+4)+(3+8)+(3+12)+(3+16)+...+(3+32)$

$3+(3+4)+(3+4(2))+(3+4(3))+...+(3+4(8))$

your function will look like this:

$f(x)=3+4(x)$, with *"x"* starting with 0 and end with 8

$\sum_{x=0}^{8} 3+4(x)$

c) $(5)+(5+2)+(5+4)+(5+6)+(5+8)+(5+10)$

This is an arithmetic sequence:

$5+(5+2)+(5+2(2))+(5+2(3))+...+(3+2(5))$

your function looks like this:

$f(x)=5+2(x)$, with *"x"* starting with 0 and end with 5

$\sum_{x=0}^{5} 5+2(x)$ OR $\sum_{x=1}^{6} 5+2(x-1)$ OR $\sum_{x=0}^{6} 3+2x$

d) $3+3(0.8)+3(0.8)^2+3(0.8)^3+3(0.8)^4+..+3(0.8)^{12}$

This is a geometic sequence:

Your function look likes this:

$f(x)=3(0.8)^x$, with *"x"* starting with 0 & end with 12

$\sum_{x=0}^{12} 3(0.8)^{x}$

e) $\sqrt{2}+2+2\sqrt{2}+4+4\sqrt{2}+....+128\sqrt{2}$

This is a geometric sequence:

$\sqrt{2}+\sqrt{2}^{2}+\sqrt{2}^{3}+\sqrt{2}^{4}+...\sqrt{2}^{15}$

The function looks like this:

$f(x)=\sqrt{2}^{x}$, with "x" starting with 1 and ending at 15

$$\sum_{x=1}^{15}\sqrt{2}^{x}$$

f) $2+\frac{2}{1.01}+\frac{2}{1.01^{2}}+\frac{2}{1.01^{3}}+.....$

This is a geometric sequence.

The function looks like this:

$f(x)=\frac{2}{1.01^{x}}$, with "x" starting with 0...

$$\sum_{x=0}^{\infty}\frac{2}{1.01^{x}}$$

g) $2-6+18-54+162-486+.....+1062882$

This is a geometric sequence:

1062882/2=531441

Log(531441)/log(3)=12

The function looks like this:

$f(x)=2(-3)^{x}$, with "x" starting at 0 and ending at 12

$$\sum_{x=0}^{12}2(-3)^{x}$$

4. Evaluate each of the following series:

a) $\sum_{n=1}^{2001}n$

$1+2+3+...2001$

$S=\frac{n}{2}(a+l)$

$S=\frac{2001}{2}(1+2001)$

$S=2003001$

b) $\sum_{k=1}^{3}\frac{1}{2k}$

$\frac{1}{2(1)}+\frac{1}{2(2)}+\frac{1}{2(3)}$

$S=\frac{11}{12}$

c) $\sum_{i=1}^{10}\frac{10}{i}$

$\frac{10}{1}+\frac{10}{2}+\frac{10}{3}+\frac{10}{4}+\frac{10}{5}+\frac{10}{6}+\frac{10}{7}+\frac{10}{8}+\frac{10}{9}+\frac{10}{10}$

This is just a series with 10 terms . It's neither arithmetic or geometric. No formulas to use.....

$S=29.29$

d) $\sum_{i=1}^{\infty}\frac{1}{5^{i}}$

$\frac{1}{5}+\left(\frac{1}{5}\right)^{2}+\left(\frac{1}{5}\right)^{3}+\left(\frac{1}{5}\right)^{4}+\left(\frac{1}{5}\right)^{5}+...$

$S=\frac{a}{1-r}$

$S=\frac{\frac{1}{5}}{1-\frac{1}{5}}$

$S=0.25$

5. Solve for "x" . Show all your work with the space provided

a) $\sum_{z=1}^{x} 5(2)^z = 1270$

$5(2)^1 + 5(2)^2 + 5(2)^3 + \ldots + 5(2)^x = 1270$

$1270 = \frac{10(1-2^n)}{1-2}$

$-1270 = 10(1-2^n)$

$-127 = 1-2^n$

$-128 = -2^n$

$128 = 2^7$

$n = 7$

$x = 7$

$\sum_{z=1}^{3} x^{z-1} = 7$

$x^{1-1} + x^{2-1} + x^{3-1} = 7$

$x^0 + x^1 + x^2 = 7$

$1 + x^1 + x^2 = 7$

b) $x(x+1) = 6$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = 2, -3$